

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [BATCH 2018-21]

B.A./B.Sc. SECOND SEMESTER (January – June) 2019

Mid-Semester Examination, March 2019

Date : 28/03/2019

MATHEMATICS (General)

Time : 11 am – 12 noon

Paper : II

Full Marks : 25

[Use a separate Answer Book for each group]

Group – A

Unit - I

1. Answer any two questions : [2×3]
- a) Show that the equation to the pair of straight lines through origin perpendicular to the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.
- b) Find the angle between the two tangents from an external point (x_1, y_1) to the circle $x^2 + y^2 = a^2$.
- c) If by orthogonal transformation, without change of origin, the expression $(ax^2 + 2hxy + by^2 + 2gx + 2fy + d)$ be changed into $(a'X^2 + b'Y^2 + 2h'XY + 2g'X + 2f'Y + D)$, then show that $a' + b' = a + b$.

Unit - II

2. Answer any one question : [1×3]
- a) Solve : $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$.
- b) Solve : $(x^3 + y^3)dx = (x^2y + xy^2)dy$.

Unit - III

3. Answer any one question : [1×4]
- a) For any triangle ABC, show that $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ with usual conventional symbols.
- b) Given $\vec{\alpha} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$. Express $\vec{\beta}$ in the form $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

Group – B

Unit - IV

- Answer any three questions : [3×4]
4. Check whether the following sequences are monotone increasing or decreasing.
- (i) $\left\{ \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} \right\}$

(ii) $\left\{ \frac{2^n}{n} \right\}$ [2+2]

5. a) Let, $\{u_n\}$ be a sequence of positive real numbers such that $\lim u_n = 0$. Does the series $\sum_{n=1}^{\infty} u_n$ is convergent? Explain it with the help of an example.

b) Check the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{n^n}{n!}$. [2+2]

6. a) State Leibnitz's test for the alternating series.

b) Does the series $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots$ is convergent? [1+3]

7. Verify Rolle's theorem for the following functions :

a) $f(x) = 2x^3 + x^2 - 4x - 2$

b) $f(x) = \tan x$ in $0 \leq x \leq \pi$. [3+1]

8. Evaluate $\int \frac{f(x)}{x^3 - 1} dx$, where $f(x)$ is a polynomial of degree 2 such that $f(0) = f(1) = 3f(2) = -3$. [4]

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